$$(\sin kx, \sin mx) = \begin{cases} \int\limits_{-\pi}^{\pi} \sin^2(kx)dx = \int\limits_{-\pi}^{\pi} \frac{1-\cos(2kx)}{2}dx = \pi & , k = m \neq 0 \\ \int\limits_{-\pi}^{\pi} \sin(kx)\sin(mx)dx = \frac{1}{2}\int\limits_{-\pi}^{\pi} (\cos(k+m)x - \cos(k-m)x)dx = 0 & , k \neq m \end{cases}$$

$$(\cos kx, \cos mx) = \begin{cases} \int\limits_{-\pi}^{\pi} 1 \cdot 1dx = 2\pi & , k = m = 0 \\ \int\limits_{-\pi}^{\pi} \cos^2(kx)dx = \int\limits_{-\pi}^{\pi} \frac{1+\cos(2kx)}{2}dx = \pi & , k = m \neq 0 \\ \int\limits_{-\pi}^{\pi} \cos(kx)\cos(mx)dx = \frac{1}{2}\int\limits_{-\pi}^{\pi} (\cos(k+m)x + \cos(k-m)x)dx = 0 & , k \neq m \end{cases}$$

$$(\sin kx, \cos mx) = \begin{cases} \int\limits_{-\pi}^{\pi} \sin(kx)\cos(mx)dx = \frac{1}{2}\int\limits_{-\pi}^{\pi} \sin(2kx)dx = 0 & , k = m \neq 0 \\ \int\limits_{-\pi}^{\pi} \sin(kx)\cos(mx)dx = \frac{1}{2}\int\limits_{-\pi}^{\pi} (\sin(k+m)x + \sin(k-m)x)dx = 0 & , k \neq m \end{cases}$$

$$(e^{-ikx}, e^{-imx}) = \int_{-\pi}^{\pi} e^{-ikx}e^{+imx}dx = \int_{-\pi}^{\pi} e^{i(m-k)x} =$$

$$= \begin{cases} \int\limits_{-\pi}^{\pi} 1dx = 2\pi & , k = m \\ \frac{1}{i(m-k)}e^{i(m-k)x}|_{-\pi}^{\pi} = \frac{1}{i(m-k)}(e^{i(m-k)\pi} - e^{-i(m-k)\pi}) = \frac{2}{m-k}\sin(m-k)\pi = 0 & , k \neq m \end{cases}$$

$k, l = 0, 1, ..., m$
$n \geq 2m + 1$

$$(\cos kx, \cos lx) = \sum_{j=1}^{n} \cos(kx_j)\cos(lx_j) =$$

$$= \begin{cases} \sum\limits_{j=1}^{n} \cos(0)\cos(0) = \sum\limits_{j=1}^{n} 1 = n & , k = l = 0 \\ \sum\limits_{j=1}^{n} \cos^2(kx_j) = \sum\limits_{j=1}^{n} \frac{1+\cos(2kx_j)}{2} = \frac{n}{2} + \frac{1}{2} \underbrace{\sum_{j=1}^{n} \cos(2kx_j)}_{0 \text{ jer } 0 \leq k \leq m \Rightarrow 0 \leq 2k \leq 2m < n} = \frac{n}{2} & , k = m \neq 0 \\ \frac{1}{2}(\underbrace{\sum_{j=1}^{n} \cos(k+l)x_j}_{0 \text{ jer } 0 \leq k,l \leq m \Rightarrow 0 \leq k+l \leq 2m < n} + \underbrace{\sum_{j=1}^{n} \cos(k-l)x_j}_{0 \text{ jer } 0 < |k-l| < n}) = 0 & , k \neq l \end{cases}$$

$$(\sin kx, \sin lx) = \sum_{j=1}^{n} \sin(kx_j)\sin(lx_j) = \begin{cases} \sum\limits_{j=1}^{n} \sin^2(kx_j) = \sum\limits_{j=1}^{n} \frac{1-\cos(2kx_j)}{2} = \frac{n}{2} & , k = l \\ \frac{1}{2}(\sum\limits_{j=1}^{n} \cos(k+l)x_j - \sum\limits_{j=1}^{n} \cos(k-l)x_j) = 0 & , k \neq l \end{cases}$$

$$(\sin kx, \cos lx) = \sum_{j=1}^{n} \sin(kx_j)\cos(lx_j) = \begin{cases} \frac{1}{2}\sum\limits_{j=1}^{n} \sin(2kx_j) = 0 & , k = l \\ \frac{1}{2}(\sum\limits_{j=1}^{n} \sin(k+l)x_j + \sum\limits_{j=1}^{n} \sin(k-l)x_j) = 0 & , k \neq l \end{cases}$$